Department of Electrical and Computer Engineering

16:332:570 ROBUST COMPUTER VISION

Homogeneous and Nonhomogeneous Objective Functions

- Least Squares, $\mathcal{J}_{LS}$, for the case $\mathbf{C}_y = \mathbf{I}_q$

$$\frac{1}{n}\sum_{i=1}^{n}(\mathbf{y}_i - \mathbf{y}_{i0})^2$$

Homogeneous and symmetric objective function.

- Least Absolute Value, $\mathcal{J}_{LAD}$, for the case $\mathbf{C}_y = \mathbf{I}_q$

$$\frac{1}{n}\sum_{i=1}^{n}|\mathbf{y}_i - \mathbf{y}_{i0}|$$

While the scalar function $\frac{1}{n}\sum_{i=1}^{n}|y_i - \alpha|$ has the location estimator $\alpha = \operatorname*{med}_i y_i$, and is robust, in general this objective function is not robust. Homogeneous and symmetric objective function.

- Least k-th Order Statistics, $\mathcal{J}_{LkOS}$,
The n residuals are sorted in ascending order, $d_{1:n} \leq d_{2:n} \ldots \leq d_{n:n}$, and the estimate is

$$d_{k:n}$$

If $k = n/2$ the least median of squares (LMedS) obtained

$$\operatorname*{med}_i d_i^2$$

This is squared residuals, but in practice we can use almost always absolute values too.
The estimators in this category are robust, *but* this is not always sufficent for computer vision. They breakdown if more than one structure is present, and/or more that 50% of the data is outlier. As we shall see later, many times in computer vision this cannot be the case. The scale parameter σ does not pay a role, only the k-th residual, because σ is a common constant in all of them. Homogeneous objective function but it is *not* symmetric.

- M-estimator, $\mathcal{J}_M$,

$$\frac{1}{n}\sum_{i=1}^{n}\rho(d_i)$$

$\rho(u)$ is a nonnegative, even-symmetric loss function, nondecreasing with $|u|$. We will be interested only in the family of *redescending* function, of the form

$$0 \leq \rho(u) \leq 1 \quad |u| \leq 1 \qquad \rho(u) = 1 \quad |u| > 1$$

and will use

$$\rho(u) = \begin{cases} 1-(1-u^2)^d & |u| \le 1 \\ 1 & |u| > 1 \end{cases}$$

most of the time with $d = 3$. The (estimated or user given) scale parameter is needed to solve the estimation. The objective function (in general!) is only symmetric but not homogeous.

- RANdom SAmple Consensus (RANSAC)

$$\max_k d_{k:n} \qquad \text{subject to} \qquad \|\delta \mathbf{y}_{k:n}\|_{\mathbf{C}_y} < s(\sigma)$$

where $s(\sigma)$ is an *user supplied* scale estimate. This estimator is used today a lot in computer vision, but the user supplied scale estimate can be a problem. We will see something else too, the projection based M-estimator (pbM-estimator), which does *not* need anything from the user. The objective function is only symmetric but not homogeous.

The Statistical Definition of Robustness

Will not be ours! Finite sample definitions.

Case I: n inliers. "typical" parameter estimate $\hat{\boldsymbol{\theta}}^{(I)}$

Case II: n_1 inliers, $n_2 = n - n_1$ *arbitrary* not structured outliers, which have to be $n_2 < 0.5n$. A lot of implicit assumptions!

$$\text{percentage of contamination} \quad \epsilon = \frac{n_2}{n}$$

and gives a parameter estimate $\hat{\boldsymbol{\theta}}^{(II)}$

Worst case bias at ϵ contamination (find the worst positions of the outliers)

$$b_M(\epsilon) = \max \left\| \hat{\boldsymbol{\theta}}^{(I)} - \hat{\boldsymbol{\theta}}^{(II)} \right\|_2$$

which have an implicit dependence on n, but only weakly.

(Explosion) breakdown point is defined as the minimum number of outliers n_2 such that

$$\epsilon_n^* = \min_{n_2} \left[\epsilon \mid b_M(\epsilon) = \infty \right]$$

and in practice instead of ∞ a very large number is enough, and the asymptotic breakdown point is used

$$\epsilon^* = \lim_{n \to \infty} \epsilon_n^*$$

Characterizes the *global* robustness of an estimator (in statistics). Under this definition $\epsilon^* \le 0.5$!

Local robustness, measured through gross error sensitivity, is worst influence a single point modified slightly can have on the estimate. LMedS (and RANSAC) have large gross error sensitivity.