Current Topics in Digital Signal Processing:

Communication Systems and Signal Processing

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Outline

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Part 1 - Brief Introduction to Digital Communication Systems:

- The basic elements of a communication system
- Channel models, channel capacity
- Digital signaling
- Receiver principles
- Wireless communication channels and fading

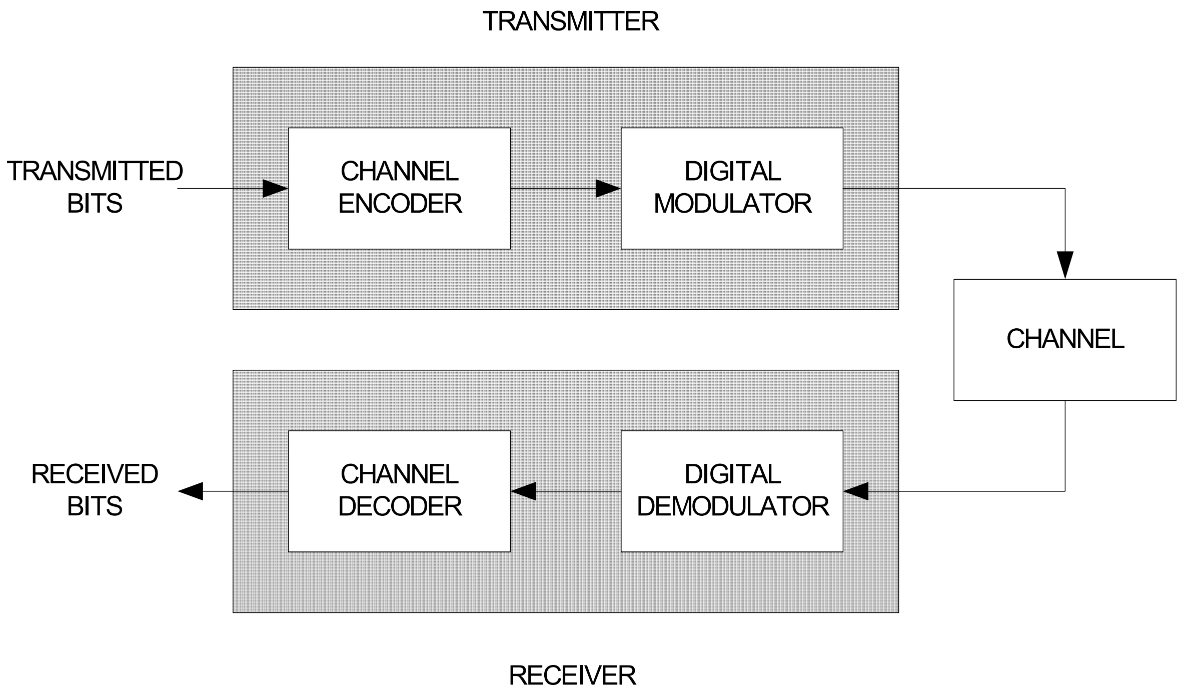
Part 2 - Multiple-Antenna Communication Systems:

- Receiver antenna diversity
- Multiple-input, multiple-output (MIMO) systems

Part 1: Brief Introduction to Digital Communication Systems

Digital Communication Systems

- **Facilitates transmission of digital information (bits) from a transmitting source to one or more receiver destinations**
- **Information that is analog by nature must be converted into a sequence of bits by a *source encoder***



Digital Communication Systems

- Channel encoder: adds redundancy to info bits to enable detection/correction of bit errors; outputs a coded bit sequence
- Digital modulator: transforms coded bit sequence into analog waveforms
- Channel: the physical communication medium
  - Distorts the signal (noise, interference, fading)
  - Distortion may cause bit errors at the receiver
  - Channel effects usually captured in equivalent baseband channel models (AWGN, Rayleigh fading, etc.)
- Digital demodulator: recovers sequence of coded bits from corrupted signal. Compensates for channel distortions
- Channel decoder: attempts to reconstruct original bit sequence with as few bit errors as possible, using knowledge of the channel code and redundancy added by channel encoder

Communication System Design

- Coding and modulation schemes must match each other as well as the channel
- Performance requirements:
  - Data rate
  - Tolerable bit error rate (BER) or frame error rate (FER)
  - Range and coverage
- Design constraints:
  - Power limitations
  - Bandwidth limitations
  - Implementation complexity → cost
  - Processing delay

Channel Capacity

- Postulated by C. L. Shannon (1948)
- Gives the upper limit on the rate at which information can be transmitted reliably:

$$C = W \log_2 (1 + \frac{P}{\sigma^2}) \text{ bits/s}$$

where W: bandwidth, P: average transmit power,
σ^2 : average noise power

- Shannon did not say how the upper limit can be attained, only that it can be attained with large complexity and delay in demodulation/decoding
- Modern communication system design is an effort to come as close to the upper limit as possible with reasonable complexity and processing delay

Digitally Modulated Signals

- **Transmitted signal:**

$$s(t) = \mathrm{Re}\{s_l(t)e^{j2\pi ft}\} \qquad s_l(t) = a(t)e^{j\theta(t)} \text{(low pass equiv.)}$$

$$s(t) = a(t)\cos(2\pi ft + \theta(t))$$

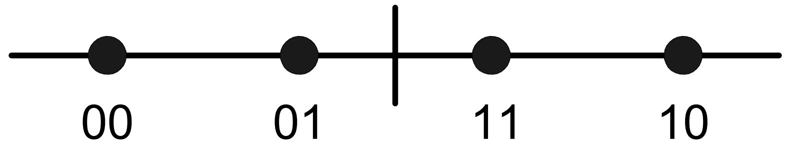
$$a(t): \text{ envelope of } s(t) \qquad \theta(t): \text{ phase of } s(t)$$

- **To transmit b bits during one signaling or symbol interval the digital modulator selects one of $M = 2^b$ distinct waveforms,** $s_i(t),\ i = 1,2,\ldots,M$
- **Each waveform is created by modulating the envelope, the phase, or both**

Modulation Examples

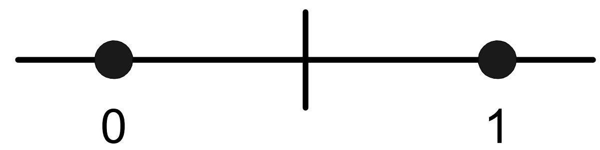
- **Pulse-amplitude modulation (PAM):** $a(t)$

EX: M**=4**

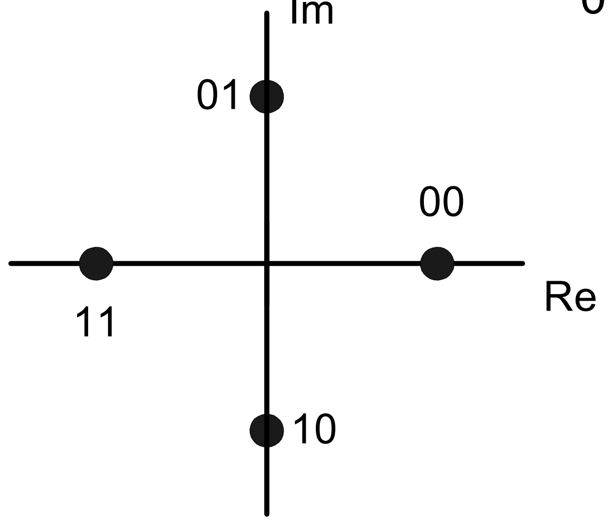


- **Phase-shift keying (PSK):** $\theta(t)$

EX: M**=2 (BPSK)**



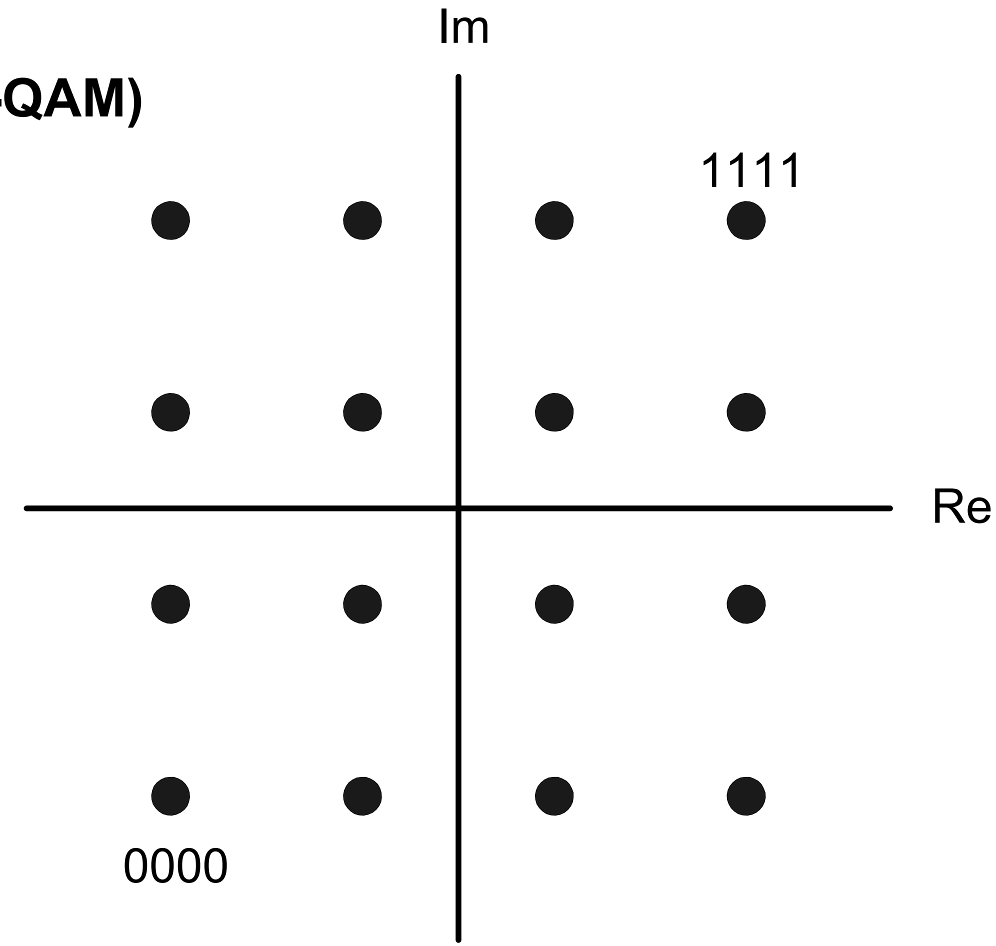
EX: M**=4 (QPSK)**



Modulation Examples

- **Quadrature amplitude modulation (QAM):** $\theta(t)$ and $a(t)$

EX: M=16 (16-QAM)



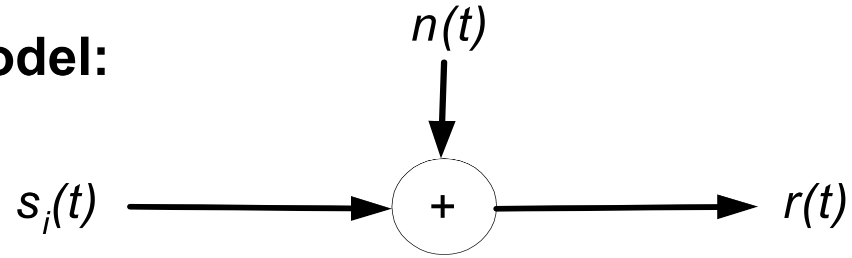
Digital Receiver for the AWGN Channel

- **AWGN = additive white Gaussian noise**
- **Transmitter sends digital information by use of M signal waveforms**
- **Received signal in the interval $0 \le t \le T$**

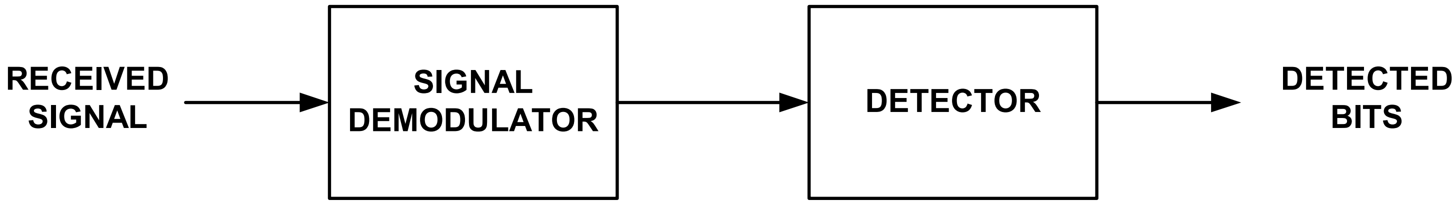
$$r(t) = s_i(t) + n(t)$$

$n(t)$: **AWGN**

- **AWGN channel model:**



Optimum Receiver



- **Demodulator: converts received waveform $r(t)$ into N-dimensional vector**

$$\mathbf{r} = [r_1 \quad r_2 \quad \ldots \quad r_N]$$

  (N: dimension of transmitted signal waveform)

- **Detector: decides which of the M possible waveforms was transmitted based on the vector $\mathbf{r}$**

Correlation-Type Demodulator

- **Signal space representation of transmitted signal:**

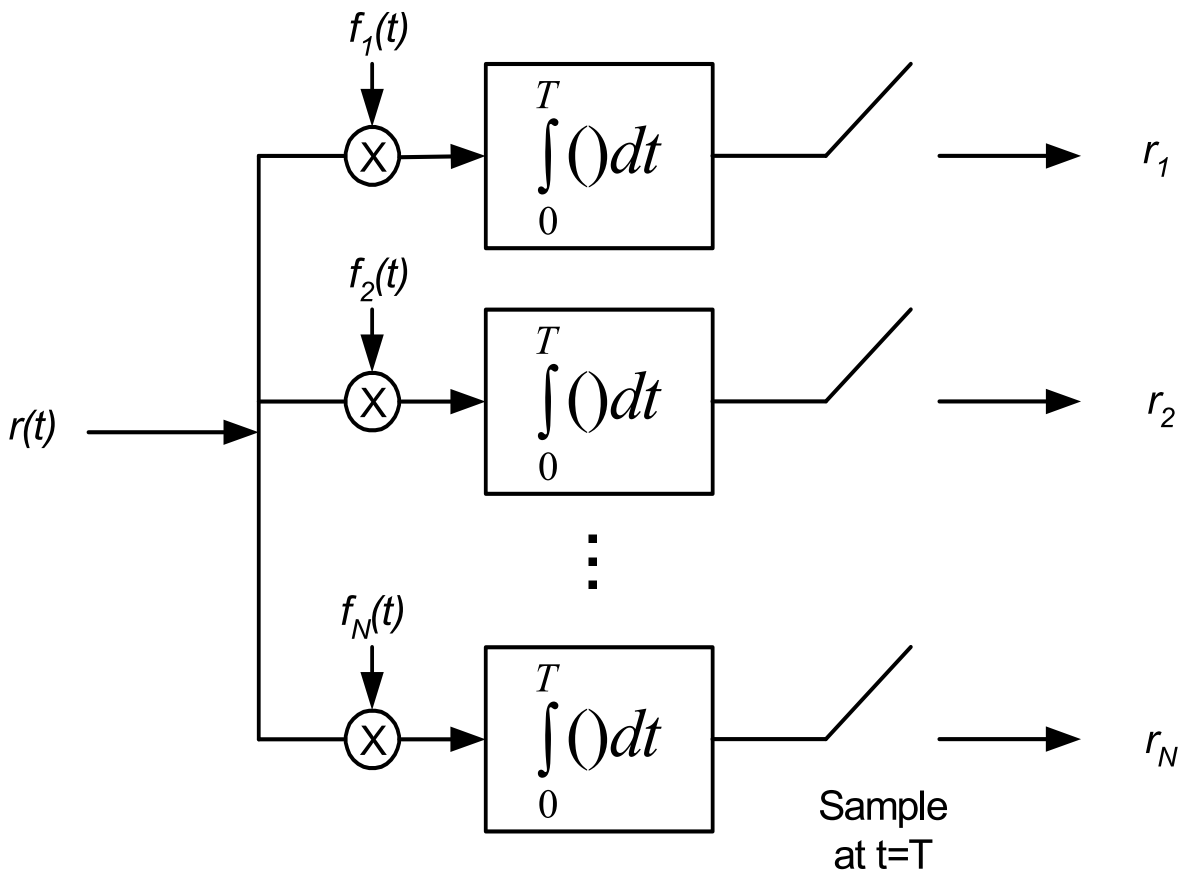
$$s_i(t) = \sum_{n=1}^{N} s_{in} f_n(t)$$

(linear combination of N basis functions $\{f_n(t)\}_{n=1}^{N}$ **)**

- **Received signal is passed through a bank of N cross correlators that compute the projection of $r(t)$ onto the N basis functions to produce the vector**

$$\mathbf{r} = [r_1 \quad r_2 \quad \ldots \quad r_N]$$

Correlation-Type Demodulator



Optimum Detector

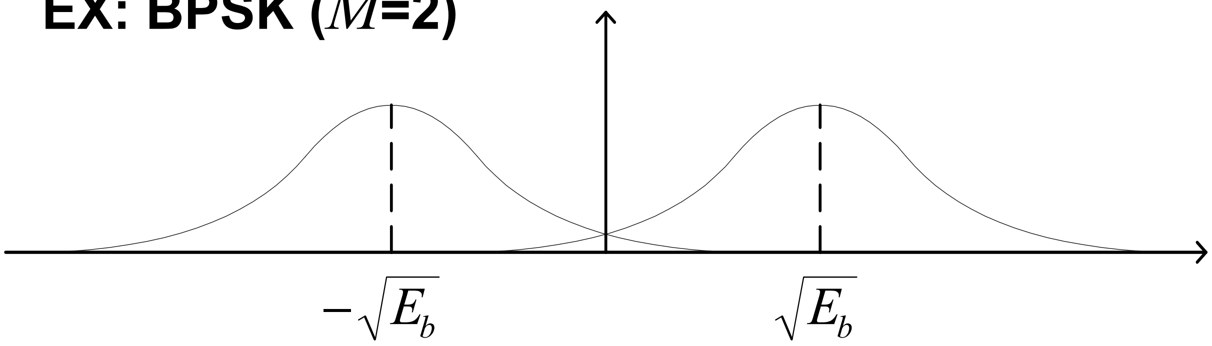
- Makes a decision on the transmitted signal based on $\mathbf{r}$ in each symbol interval such that the probability of a correct decision is maximized:

$$\max_{i=1,2,\ldots,M} \{\Pr(\text{signal } s_i \text{ was transmitted} \mid \mathbf{r})\}$$

- $b = \log_2 M$ bits are associated with each signal (symbol) decision

Performance of Optimum Receiver

EX: BPSK (M=2)



$N = 1$ dimension

$r = s_i + n$

$s_1 = \sqrt{E_b},\ s_2 = -s_1$

Noise power : $\sigma^2 = N_0 / 2$

Probability density function (pdf) of r:

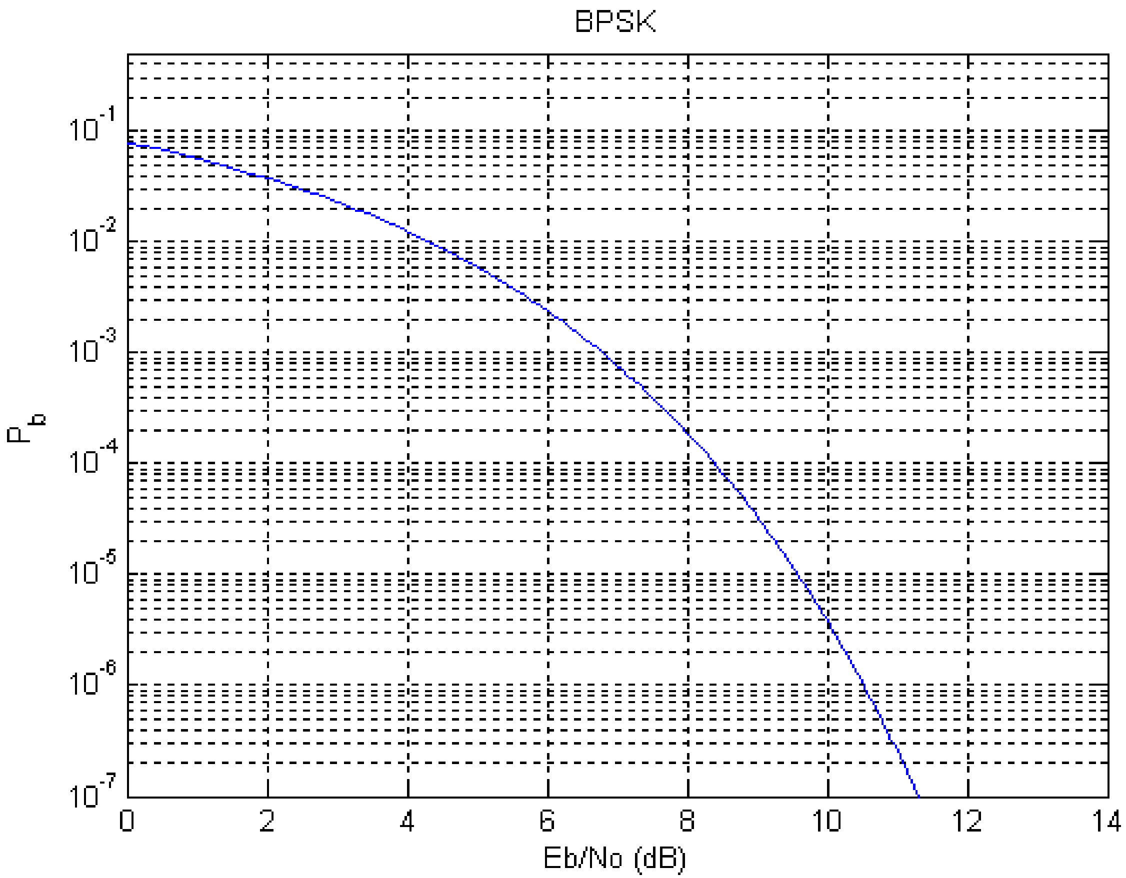
$$p(r \mid s_1) = \frac{1}{\sqrt{\pi N_0}} e^{-(r-\sqrt{E_b})^2 / N_0}, \quad p(r \mid s_2) = \frac{1}{\sqrt{\pi N_0}} e^{-(r+\sqrt{E_b})^2 / N_0}$$

$$\Pr(error \mid s_1) = \int_{-\infty}^{0} p(r \mid s_1) dr = Q(\sqrt{\frac{2E_b}{N_0}}) = \Pr(error \mid s_2)$$

$$P_b = 1/2 \Pr(error \mid s_1) + 1/2 \Pr(error \mid s_2) = Q(\sqrt{\frac{2E_b}{N_0}})$$

Bit Error Probability in AWGN

Bit error probability as a function of signal-to-noise ratio:



Communication Through Wireless Channels

Distortions:

- Multipath propagation → may cause inter-symbol interference (ISI)
- Signal fading
  - Due to moving transmitter/receiver as well as moving objects in the transmission path
  - Multiplicative process
  - Often modeled as Rayleigh distributed
- Scattering
- Shadowing
- Path loss
- Noise – modeled as AWGN
- Interference from other transmissions
- Interference from electrical equipment

Fading Channels

- **Model for received signal (low pass equivalent):**

$$r(k) = r(t = kT) = h(k)s(k) + n(k),\ 0 \le t \le T$$

$$h(k) = \alpha(k)e^{j\varphi(k)}$$

Complex valued channel coefficient

- **Slow fading: fading constant during at least one symbol interval**
  - **Channel gain:** $\alpha(k) = \alpha$
  - **Phase shift:** $\varphi(k) = \varphi$

- **Average signal-to-noise ratio (BPSK):**

$$\bar{\gamma}_b = \frac{E_b}{N_0} E[\alpha^2]$$

Bit Error Probability

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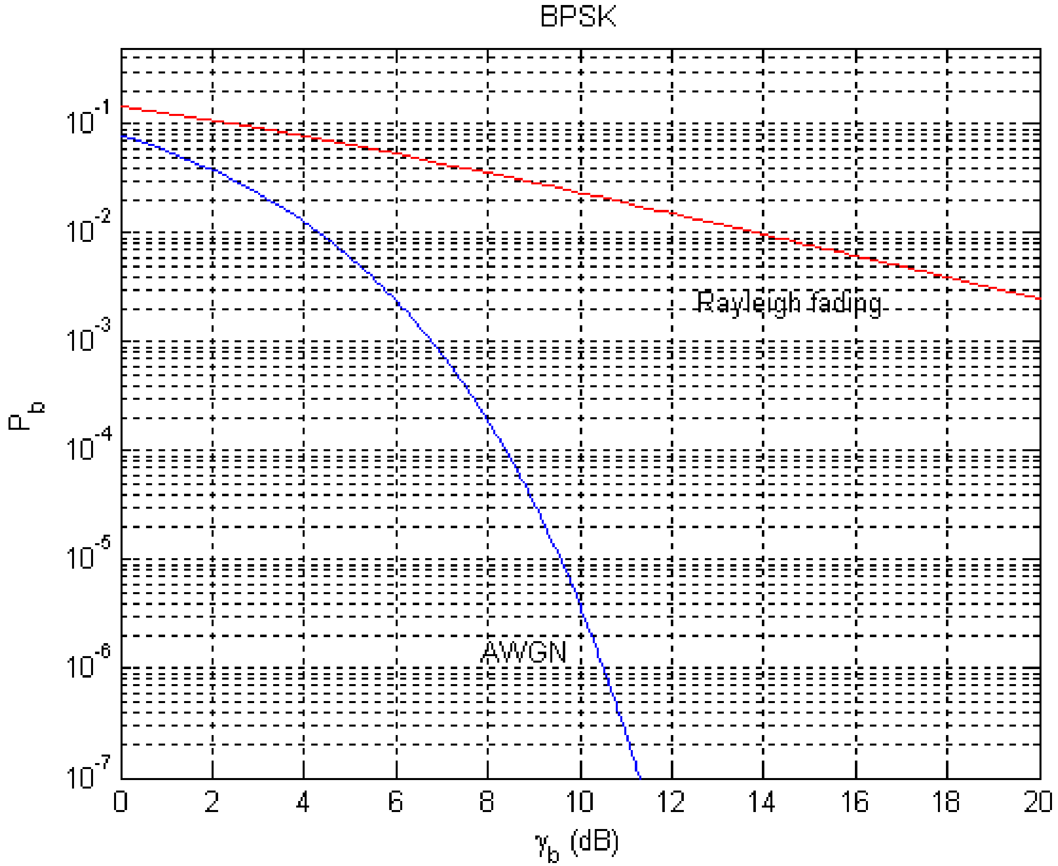
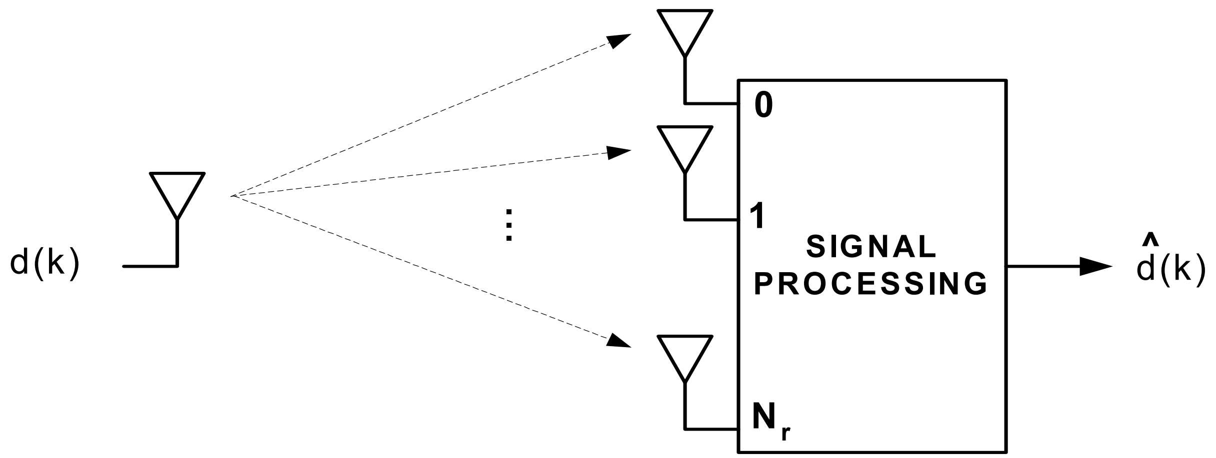
Bit error probability vs SNR for AWGN and Rayleigh fading channels:



$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right)$$

Part 2: Multiple-Antenna Systems

Receiver Antenna Diversity



- **Received signals:**

$$r_i(k) = h_i(k)d(k) + n_i(k)$$

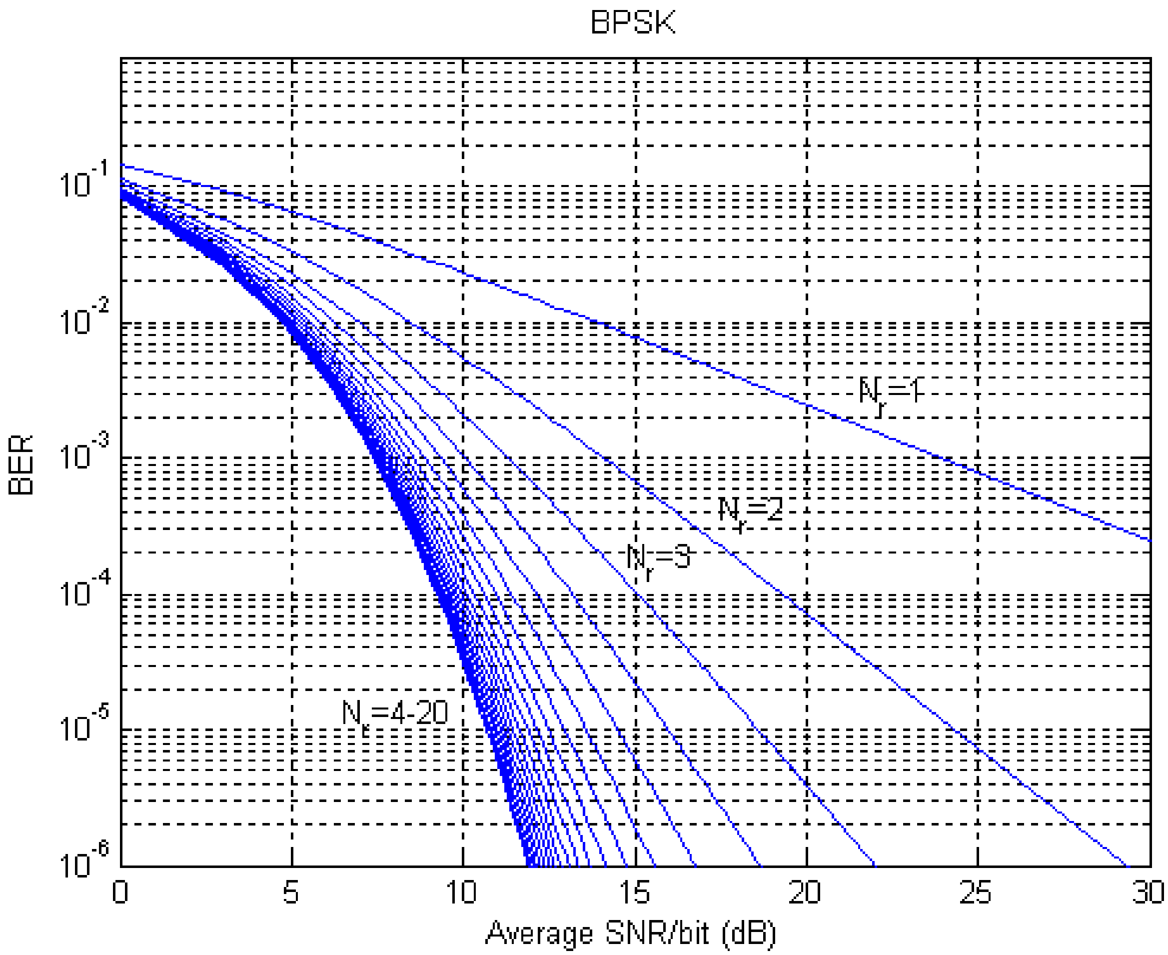
- **"Maximal Ratio Combining":**

$$y(k) = \sum_{i=1}^{N_r} h_i^*(k) r_i(k) = \sum_{i=1}^{N_r} |h_i(k)|^2 \, d(k) + \tilde{n}(k)$$

- **Symbol estimate:**

$$\hat{d}(k) = (1 / \sum_{i=1}^{N_r} |h_i^*(k)|^2) y(k)$$

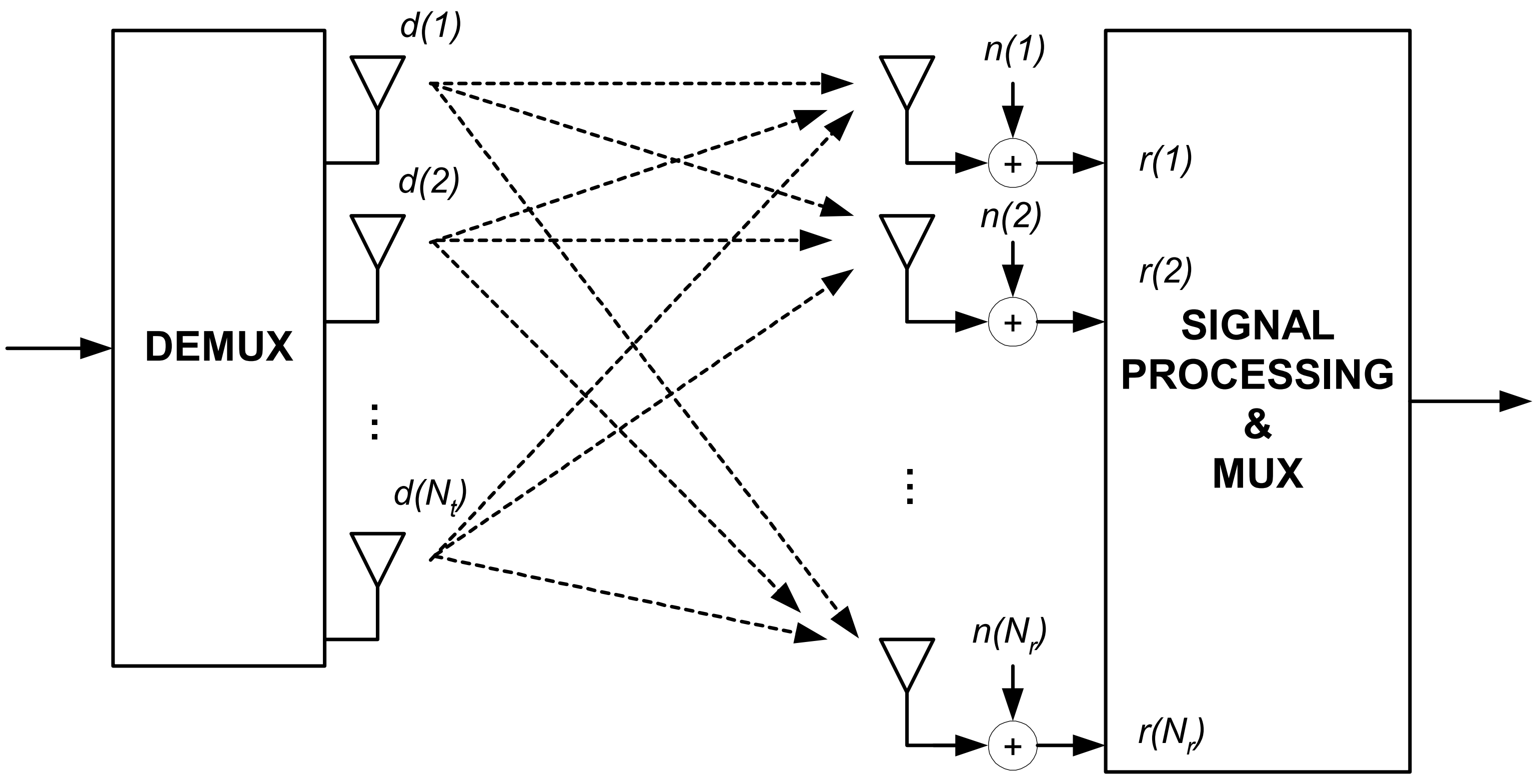
Performance with Diversity



MIMO Systems

- MIMO = multiple-input, multiple-output – refers to a system with multiple transmit and multiple receive antennas
- How?
  - Multiple antennas facilitate multiple parallel spatial channels → spatial multiplexing
  - Sufficiently rich scattering
  - High SNR
- Why MIMO?
  - Increased data rate (scales roughly with no. of antennas)
  - Improved performance
  - Increased range and coverage
  - Improved quality of service

MIMO Channel Model



Narrowband MIMO matrix channel model: $\mathbf{r}=\mathbf{Hd}+\mathbf{n}$

MIMO Channel Model

- **Narrowband MIMO discrete-time memoryless channel (flat fading):**

$$\begin{bmatrix} r(1) \\ r(2) \\ \vdots \\ r(N_r) \end{bmatrix} = \begin{bmatrix} h(1,1) & h(1,2) & \cdots & h(1,N_t) \\ h(2,1) & h(2,2) & \cdots & h(2,N_t) \\ \vdots & \vdots & \ddots & \vdots \\ h(N_r,1) & h(N_r,2) & \cdots & h(N_r,N_t) \end{bmatrix} \begin{bmatrix} d(1) \\ d(2) \\ \vdots \\ d(N_t) \end{bmatrix} + \begin{bmatrix} n(1) \\ n(2) \\ \vdots \\ n(N_r) \end{bmatrix}$$

$$h(i,j):\ \mathrm{N}(0,\sigma_h{}^2)\ \text{(Complex Gaussian)},\quad |h(i,j)|^2:\ \text{Rayleigh}$$

$$n(i):\ \mathrm{N}(0,\sigma^2)\ \text{(Complex Gaussian)}$$

- **Eigen-decomposition of channel matrix $\mathbf{H}$:**
  - $\mathbf{H} = \mathbf{UDV}^{\mathbf{H}}$
  - **$\mathbf{U}$, $\mathbf{V}$ are unitary matrices (I.e., $\mathbf{U}^{\mathbf{H}}\mathbf{U}=\mathbf{I}$)**
  - **$\mathbf{D}$ is a diagonal matrix of eigenvalues of $\mathbf{R}=\mathbf{H}^{\mathbf{H}}\mathbf{H}$**

Capacity of MIMO Channels

$$C = \log_2 \left| \mathbf{I} + \frac{1}{\sigma^2} \mathbf{H}^H \mathbf{H} \right| = \sum_{i=1}^{N} \log_2 (1 + \lambda_i \cdot SNR_i) \quad \text{bits/s/Hz}$$

$$SNR_i = \frac{P_i}{\sigma_i} \quad \text{and} \quad N = \text{rank}(\mathbf{H}) = \min(N_t, N_r)$$

Example 1: 2x2 line-of-sight AWGN channel with $P_i = P$

- **Channel matrix:** $\mathbf{H} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
- **Eigenvalues:** $\mathbf{D} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$
- **Capacity:**

$$C = \log_2 (1 + \frac{4P}{\sigma^2})$$

Capacity of MIMO Channels

Example 2: 2x2 flat Rayleigh fading channel with $P_i = P$

- **Channel matrix:**

$$\mathbf{H} = \begin{bmatrix} h(1,1) & h(1,2) \\ h(2,1) & h(2,2) \end{bmatrix}, \quad h(i,j): \mathrm{N}(0,1/2) \ \text{(Normalized)}$$

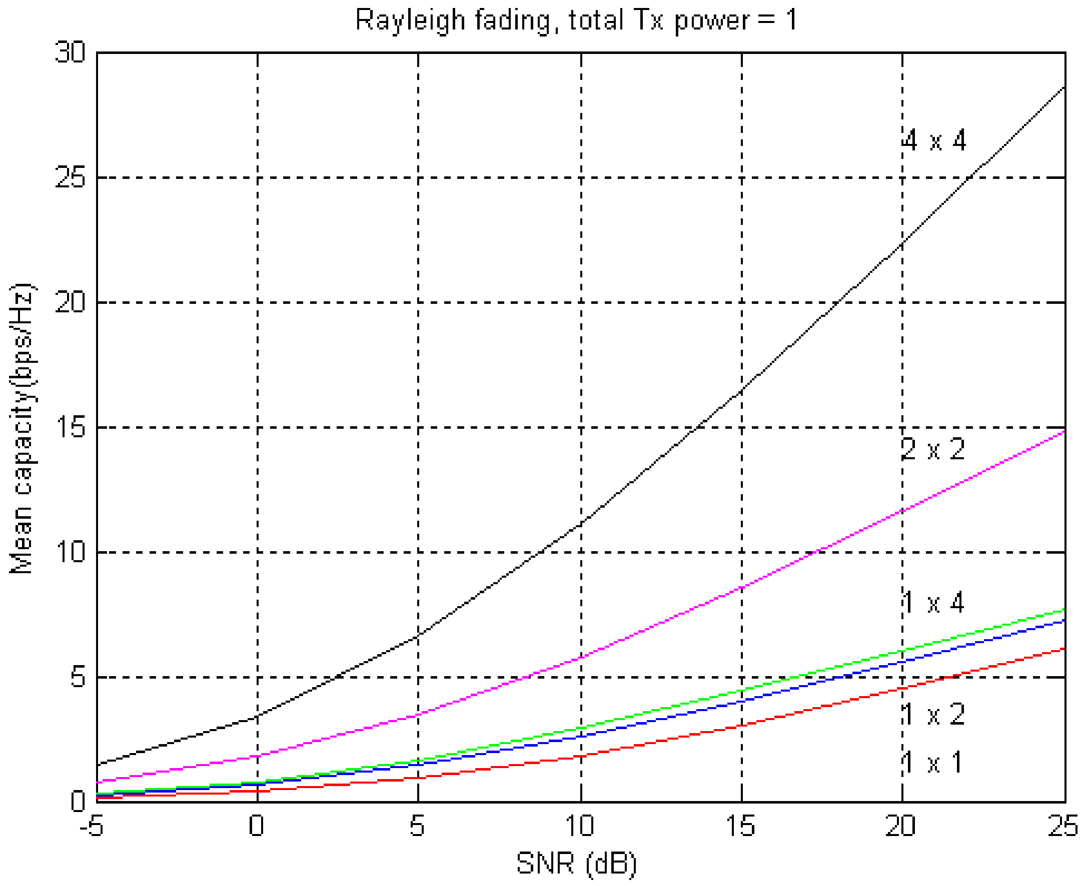
- **Eigenvalues:**

$$\mathbf{D} = \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix}, \quad \mathrm{E}[\lambda_1] = 3.5, \ \ \mathrm{E}[\lambda_2] = 0.5$$

- **Average capacity:**

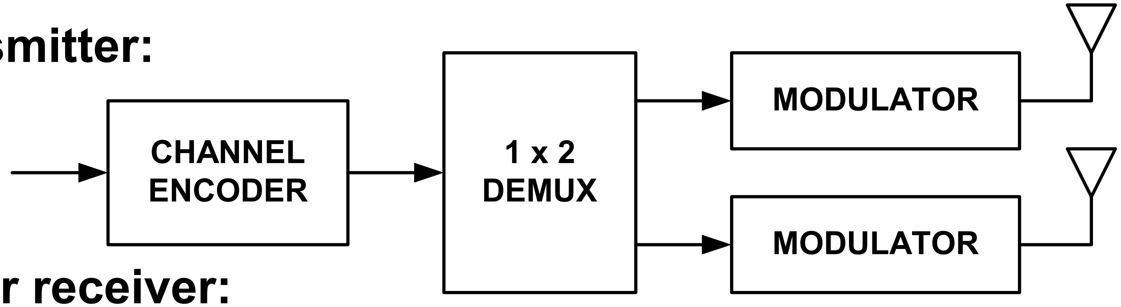
$$\bar{C} = \log_2(1 + \frac{3.5P}{\sigma^2}) + \log_2(1 + \frac{0.5P}{\sigma^2})$$

Mean Capacity of MIMO & SIMO

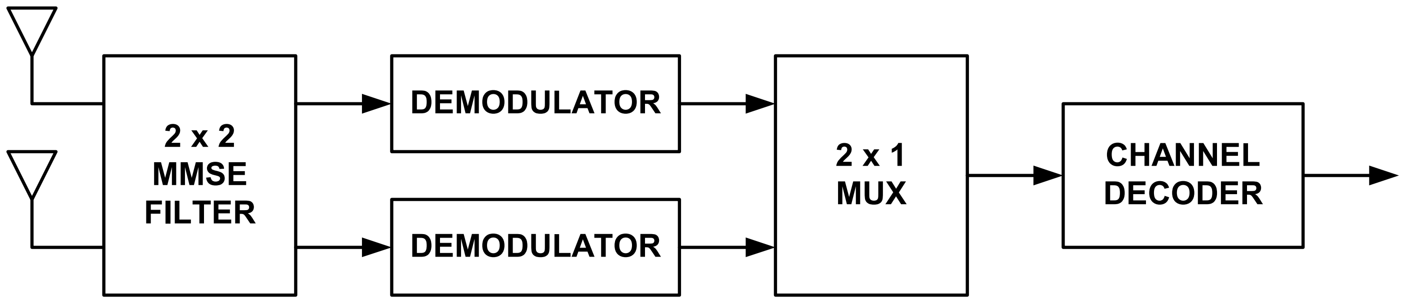


MIMO Transmit and Receive Processing

- **(2x2) Transmitter:**



- **(2x2) Linear receiver:**



  - Pilot signals used to estimate $\mathbf{H}$ at the receiver
  - 2 Rx antennas can null 1 signal (“interferer”) at a time

- **Receiver estimates SNRs per channel based on $\mathbf{H}$ and feeds them back to transmitter; transmitter selects data rate (i.e., coding + modulation) based on receive SNRs (“partial channel state information” scheme)**

Some References

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- J. G. Proakis, "Digital Communications", McGraw-Hill, 4th edition, 2001
- J.G. Proakis and M. Salehi, "Contemporary Communication Systems Using Matlab", PWS, 1998
- T. S. Rappaport, "Wireless Communications", Prentice-Hall, 1999